U.S. DEPARTMENT OF COMMERCE
National Technical Information Service
PB80-128903

Gravimetric Tidal Loading Computed from Integrated Green's Functions

(U.S.) National Geodetic Survey, Rockville, MD

Oct 79

79113002

PB80-128903

NOAA Technical Memorandum NOS NGS 22

GRAVIMETRIC TIDAL LOADING COMPUTED FROM INTEGRATED GREEN'S FUNCTIONS

Rockville, Md.
October 1979

noaa NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION / National Ocean Survey

NOAA Technical Publications

National Ocean Survey/National Geodetic Survey subseries

The National Geodetic Survey (NGS) of the National Ocean Survey (NOS), NOAA, establishes and maintains the basic National horizontal and vertical networks of geodetic control and provides governmentwide leadership in the improvement of geodetic surveying methods and instrumentation, coordinates operations to assure network development, and provides specifications and criteria for survey operations by Federal, State, and other agencies.

NGS engages in research and development for the improvement of knowledge of the figure of the Earth and its gravity field, and has the responsibility to procure geodetic data from all sources, process these data, and make them generally available to users through a central data base.

NOAA Technical Memorandums and some special NOAA publications are sold by the National Technical Information Service (NTIS) in paper copy and microfiche. Orders should be directed to NTIS, 5285 Port Royal Road, Springfield, VA 22161 (telephone: 703-557-4650). NTIS customer charge accounts are invited; some commercial charge accounts are accepted. When ordering, give the NTIS accession number (which begins with PB) shown in parentheses in the following citations.

Paper copies of NOAA Technical Reports, which are of general interest to the public, are sold by the Superintendent of Documents, U.S. Government Printing Office (GPO), Washington, DC 20402 (telephone: 202-783-3238). For prompt service, please furnish the GPO stock number with your order. If a citation does not carry this number, then the publication is not sold by GPO. All NOAA Technical Reports may be purchased from NTIS in hard copy and microform. Prices for the same publication may vary between the two Government sales agents. Although both are nonprofit, GPO relies on some Federal support whereas NTIS is self-sustained.

An excellent reference source for Government publications is the National Depository Library program, a network of about 1,300 designated libraries. Requests for borrowing Depository Library material may be made through your local library. A free listing of libraries currently in this system is available from the Library Division, U.S. Government Printing Office, 5236 Eisenhower Ave., Alexandria, VA 22304 (telephone: 703-557-9013).

NOAA geodetic publications

Classification, Standards of Accuracy, and General Specifications of Geodetic Control Surveys. Federal Geodetic Control Committee, John O. Phillips (Chairman), Department of Commerce, NOAA, NOS, 1974 reprinted annually, 12 pp (PB265442). National specifications and tables show the closures required and tolerances permitted for first-, second-, and third-order geodetic control surveys. (A single free copy can be obtained, upon request, from the National Geodetic Survey, C13x4, NOS/NOAA, Rockville MD 20852.)

Specifications To Support Classification, Standards of Accuracy, and General Specifications of Geodetic Control Surveys. Federal Geodetic Control Committee, John O. Phillips (Chairman), Department of Commerce, NOAA, NOS, 1975, reprinted annually, 30 pp (PB261037). This publication provides the rationale behind the original publication, "Classification, Standards of Accuracy, ..." cited above. (A single free copy can be otained, upon request, from the National Geodetic Survey, C13x4, NOS/NOAA, Rockville MD 20852.)

NOAA Technical Memorandums, NOS/NGS subseries

NOS NGS-1 Use of climatological and meteorological data in the planning and execution of National Geodetic Survey field operations. Robert J. Leffler, December 1975, 30 pp (PB249677). Availability, pertinence, uses, and procedures for using climatological and meteorological data are discussed as applicable to NGS field operations.

NOS NGS-2 Final report on responses to geodetic data questionnaire. John F. Spencer, Jr., March 1976, 39 pp (PB254641). Responses (20%) to a geodetic data questionnaire, mailed to 36,000 U.S. land surveyors, are analyzed for projecting future geodetic data needs.

NOS NGS-3 Adjustment of geodetic field data using a sequential method. Marvin C. Whiting and Allen J. Pope, March 1976, 11 pp (PB253967). A sequential adjustment is adopted for use by NGS field parties.

NOS NGS-4 Reducing the profile of sparse symmetric matrices. Richard A. Snay, June 1976, 24 pp (PB-258476). An algorithm for improving the profile of a sparse symmetric matrix is introduced and tested against the widely used reverse Cuthill-McKee algorithm.

(Continued at end of publication)

NOAA FORM 25-13 (1-78)	BIBLIOGRAPHIC DATA SHEET	U. S. DEPARTMENT OF COMMERCE NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION

1. NOAA ACCESSION NUMBER NOAA-79113002	2.	3. RECIPIENT'S ACCESSION NUMBER PB80-128903
4. TITLE AND SUBTITLE Gravimetric Tidal Loading Computed from Integrated Green's Functions		5. REPORT DATE Oct 1979
		6.
7. AUTHOR(S) Clyde C. Goad		8. REPORT NO. NOAA-TM-NOS-NGS-22
9. PERFORMING ORGANIZATION NAME AND ADDRESS NOAA, National Ocean Survey, Rockville, MD 20852, National Geodetic Survey		10. PROJECT/TASK NO.
		11. CONTRACT/GRANT NO.
12. SPONSORING ORGANIZATION NAME AND ADDRESS Same		13. TYPE OF REPORT AND PERIOD COVERED Tech. Memo.
		14.

15. PUBLICATION REFERENCE

NOAA Technical Memorandum NOS NGS 22, October 1979. 18 p, 3 tab, 17 ref.

16. ABSTRACT

The usual method of predicting the effects of ocean tides on geodetic measurements is to use impulse response functions (called Green's functions) by convolving them with the desired ocean tide model. Because ocean tide representations are usually expressed as areas or cells of constant amplitude and phase, it has been found that the integrals of Green's functions are more desirable for use with tidal loading calculations. Predictions are presented of the loading effects on Earth-tide gravimeter measurements using a global M2 ocean tide model developed by E. W. Schwiderski. Souriau's calculations for loading effects on tidal gravity data available for western Europe are confirmed to an accuracy of ±0.2 microgal for most sites. Other results are available from California, Australia, and Japan. (Author)

17. KEY WORDS AND DOCUMENT ANALYSIS

17A. DESCRIPTORS

*Ocean tides, *Geodetic surveys, Green's functions

17B. IDENTIFIERS/OPEN-ENDED TERMS

Tidal loading

17C. COSATI FIELD/GROUP

8E, 12A

18. AVAILABILITY STATEMENT Released for distribution: Elaine S. Downs	19. SECURITY CLASS (This report) UNCLASSIFIED	21. NO. OF PAGES 23 p.
	20. SECURITY CLASS (This report) UNCLASSIFIED	22. PRICE A02 - A01

NOAA FORM 25-13 (1-78) SUPERSEDES ALL PREVIOUS EDITIONS.

NOAA Technical Memorandum NOS NGS 22

GRAVIMETRIC TIDAL LOADING COMPUTED FROM INTEGRATED GREEN'S FUNCTIONS

Clyde C. Goad

Rockville, Md.
October 1979

UNITED STATES DEPARTMENT OF COMMERCE
Juanita M. Kreps, Secretary
/ NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
Richard A. Frank, Administrator
/ National Ocean Survey
Herbert R. Lippold, Jr., Director

CONTENTS

Gravimetric Tidal Loading Computed from Integrated Green's Functions

Clyde C. Goad
National Geodetic Survey
National Ocean Survey, NOAA
Rockville, Md. 20852

ABSTRACT. The usual method of predicting the effects of ocean tides on geodetic measurements is to use impulse response functions (called Green's functions) by convolving them with the desired ocean tide model. Because ocean tide representations are usually expressed as areas or cells of constant amplitude and phase, it has been found that the integrals of Green's functions are more desirable for use with tidal loading calculations.

Predictions are presented of the loading effects on Earth-tide gravimeter measurements using a global M2 ocean tide model developed by E. W. Schwiderski. Souriau's calculations for loading effects on tidal gravity data available for western Europe are confirmed to an accuracy of ±0.2 microgal for most sites. Other results are available from California, Australia, and Japan.

INTRODUCTION

For several decades prediction of the response of the Earth to variable loads has captured the interest of geophysicists, oceanographers, and geodesists. During the past decade, improvements in Earth modeling enabled researchers to determine realistic response functions given the elastic properties of the Earth. It is only natural to use these tools to predict the response of the Earth to the loading of the ocean tides. In this paper an alternate form has been developed for the response functions. This method requires that the ocean tide height be given in terms of areas of constant amplitude and phase, the way global

tide models are usually computed. When such representations of the tide height are available over any cell or area, the amplitude times the sine or cosine of phase is constant and can be taken outside the integral. Then the integral of the impulse response function (or Green's function) remains to be evaluated. This integral is stable (by removal of one singularity) and enables one easily to use any set of load deformation coefficients in the study of Earth gravity or displacement response. Previously, one had to use published Green's functions (Farrell 1972) or compute the functions, requiring the evaluation of infinite series which do not converge when the angular argument is small. The technique presented here also allows one properly to account for the distance of the instrument above sea level. Of course, this method is not restricted to global representations. It also can be used in any region where the tide is represented by areas of constant amplitude and phase.

The results presented in this paper are based on the global representation of the M2 tide developed by E. W. Schwiderski of the Naval Surface Weapons Center, Dahlgren, Va. This model is constrained to agree with coastal and island tide data, and is used here to predict the effect of ocean loading on gravity data taken at several areas of the Earth. The results look promising. For example, in Australia, the tidal perturbations caused by the oceans appear to be predicted to the 0.5-microgal level, slightly better than the 1-microgal level reported by Breteger and Mather (1978). When the tidal gravimeter stations are more than 1° from the land-water boundaries, excellent agreement is demonstrated with Souriau (1979) who used 0.25° grids for nearby seas when correcting many tidal gravity observations taken in western Europe.

RESPONSE FUNCTIONS

Loading Potential

For the special case of the ocean tides, let the mass distribution be represented by a constant density layer with varying height, h, covering a large sphere of radius, a. The gravitational potential becomes

$$U' = G\rho a^2 \iint \frac{h(\theta,\alpha)\ \sin\theta\ d\theta\ d\alpha}{\sqrt{a^2 + r^2 - 2ar\cos\theta}} . \tag{1}$$

The quantities θ and α are the central angle and azimuth, r is the distance from the center of the Earth, G is the gravitational constant, and ρ is the density of sea water. Noting the presence of the generating function for the Legendre polynomials $P_n(\cos\theta)$, the integral becomes

$$U_e' = G\rho a^2 \sum_{n=0}^{\infty} \iint h(\theta,\alpha)\ P_n(\cos\theta)\ \frac{a^n}{r^{n+1}}\ \sin\theta\ d\theta\ d\alpha \tag{2a}$$

for the solution outside the sphere (exterior), and

$$U_i' = G\rho a^2 \sum_{n=0}^{\infty} \iint h(\theta,\alpha)\ P_n(\cos\theta)\ \frac{r^n}{a^{n+1}}\ \sin\theta\ d\theta\ d\alpha \tag{2b}$$

for the interior solution. The solution at the spherically coated surface is obtained from eq. (1) by letting r = a. However, measurements with Earth tidal gravimeters normally take place above sea level, and thus eq. (1) or (2a) must be used.

Newtonian Tidal Attraction Evaluation

Several numerical solutions of the Laplace tidal equations have recently become available. Previously, solutions were given by cotidal charts which showed contours of constant amplitude and phase, or low degree spherical harmonic expansions. Now the solutions are almost always provided in gridded form where the tidal amplitude and phase are given as constants over small areas (e.g., 1° geographic squares). Let A_i and σ_i be the amplitude and Greenwich phase, respectively, over the i-th region. Then the tide height, $h_i(t)$, at time t is given by

$$h_i(t) = A_i \cos(\bar{n} \cdot \bar{\beta} + \sigma_i) \tag{3}$$

where $\bar{n}$ is the coefficient vector for a given constituent and $\bar{\beta}$ is a vector of six astronomical angles (Cartwright and Tayler 1971). Decomposing eq. (3) with multiple angle identities yields the sinusoidal and cosinusoidal terms

$$h_i(t) = h_i^c \cos(\bar{n} \cdot \bar{\beta}) - h_i^s \sin(\bar{n} \cdot \bar{\beta}). \tag{4}$$

Substituting eq. (4) into eq. (1), and assuming that the h_i^c and h_i^s are constant over the i-th sector bounded by azimuthal angles α_{1i} and α_{2i} and central angles θ_{1i} and θ_{2i}, yields a rather simple representation of the direct loading potential

$$U' = \sum_i \frac{G\rho a}{r} (\alpha_{2i} - \alpha_{1i}) \left[h_i^c \cos(\bar{n} \cdot \bar{\beta}) - h_i^s \sin(\bar{n} \cdot \bar{\beta}) \right] \cdot \left[\sqrt{a^2 + r^2 - 2ar\cos\theta} \right]_{\theta_{1i}}^{\theta_{2i}} . \tag{5}$$

Differentiation of eq. (5), or either 2a or 2b, with respect to r yields gravity above or below the tidal layer. The

mathematical model of an infinitesimally thin layer covering the surface of a sphere exhibits a discontinuity in gravity as one crosses the boundary of the tidal sheet. Thus gravity readings made very close to coasts or on islands can be significantly perturbed. (See Pekeris (1978) for a further discussion of the problem.)

Load Deformation Coefficients h_n', k_n', and l_n'

Because the Earth is not a perfectly rigid body, a deformation will occur due to the application of the tidal load. The response of the Earth to the load is described by the load deformation coefficients h_n', k_n', l_n' (Munk and MacDonald 1960). Let U_n' represent the n-th degree contribution to the loading potential in eq. (2). Then the change in the distance from the center of the Earth (or height) is given by $h_n'U_n'/g$. The change in the n-th degree potential caused by the redistribution of mass is represented by the coefficients k_n'. The actual potential after deformation is given by $(1+k_n')U_n'$. Similarly to h_n', the l_n' represents horizontal displacements of degree n. Numerical values of load deformation coefficients used in this study have been taken from Farrell (1972) and Zschau (1978).

Gravity Change Caused by Deformation

Since the direct or Newtonian contribution to gravity has been given, one must now determine the contribution to gravity resulting from deformation. For the external problem differentiation of $\Sigma k_n'U_n'$ with respect to r gives

$$-\frac{1}{r} \sum_{n=o}^{\infty} k_n' \,(n+1)\, U_n'.$$

The change in gravity resulting from height changes is given by

$$\frac{2g}{r} \sum_{n=o}^{\infty} h_n' \, U_n'/g.$$

The combination of these two terms along with the Newtonian contribution gives the total change in gravity caused by the tidal mass layer

$$\Delta g_{tide} = \frac{1}{r} \sum_{n=o}^{\infty} \left[(n+1)(1+k_n') - 2h_n' \right] U_n' \qquad (6)$$

where the sign is reversed so a downward attraction is positive (as is measured by a gravimeter). The term for the attraction inside the brace should be used rather than the corresponding term for attraction under the tidal sheet given by Farrell (1972) and Longman (1963).

Summation of Series

As previously discussed, the Newtonian contribution is best evaluated by using eq. (5) which avoids the evaluation of an in finite sum. The important difference between this approach and that given by other investigators is that the integrals of Green's functions are computed rather than Green's functions themselves. Again noting that the amplitude and phase of the ocean tide are constant over limited areas (as was done for the Newtonian attraction), the tide height term can be taken outside the integral to simplify the process. This technique then reduces to an evaluation of the expression $\int P_n(\cos\theta)\sin\theta\, d\theta$ which can be obtained by using recursive expressions. Let $T_n(\theta) = \int P_n(\cos\theta)\sin\theta\, d\theta$. Then the T's are given by

$$T_n(\theta) = \frac{\sin\theta}{n(n+1)} P_{n1}(\cos\theta) \qquad (7)$$

where $P_{n1}(\cos\theta)$ is the associated Legendre function of degree n and order one. The recursive expressions for $P_{n1}(\cos\theta)$ are used to obtain

$$T_n(\theta) = \frac{2n-1}{n+1} \cos\theta \; T_{n-1}(\theta) - \frac{n-2}{n+1} \; T_{n-2}(\theta). \qquad (8)$$

The functions T_n are very desirable in that the infinite sum $\sum_{n=0}^{\infty} T_n$ exhibits no singularities as does $\sum_{n=0}^{\infty} P_n$ when the central angles are small. These are essentially the disk factors that Farrell (1972, 1973) used to improve the convergence characteristics of Green's functions. Although not required, Kummer's method (Farrell 1972) can also be used to facilitate the evaluation of the infinite series because h_n' and nk_n' approach constants as n gets large. The terms involving height above sea-level, $(a/r)^n$, remain in the infinite sums. These can be important especially if the gravimeter is placed in rather high locations. Pekeris (1978) has shown that as one approaches the surface from above or below the tidal sheet, the infinite sum becomes a composition of two terms. One term represents the solution in the center of the surface or boundary, and the other is a delta function accounting for the attraction of the mass directly above or below.

The technique of using the integral of Green's functions rather than Green's functions themselves does not limit itself to gravity calculations only. The same technique can be used for all effects such as displacement, tilt, and strain calculations to remove one singularity. It is not only limited to global tide models, but can be used regionally if the regional representations are given as areas of constant amplitude and phase.

All ocean tidal contributions calculated in this study omitted the degree zero term (n=0) in order to impose mass conservation.

Because some of the observations were taken near the ocean, a small nonzero initial central angle was used.

M2 OCEAN TIDE

The global 1° square representation of the M2 constituent was obtained from E. W. Schwiderski, of the Naval Surface Weapons Center, Dahlgren. This M2 model was generated by "hydrodynamical interpolation" (Schwiderski 1978). That is, in solving the Laplace tidal equations, more than 2,000 empirical tide gage observations from continental and island stations were used to constrain the solution height amplitude and phase inside the grid compartments where tide gage observations were available. This feature is very important for studying ocean loading. Frequently, modification of global tide models is undertaken using more realistic local models because gravity measurements taken near coasts are sensitive to the local tide. However, for the results quoted here, no modifications of the Schwiderski M2 model were made. The fine mesh size (1°x1°) was also an important consideration in choosing this particular model for this study.

RESULTS

Tidal Observations

Normally, analyses of tidal gravity series are given in terms of amplitude factor δ and phase ψ. The amplitude factor is the ratio of actual tidal response amplitude to the theoretical gravity value for a rigid Earth. Solutions of Earth modeling yield an amplitude factor of δ=1.16 and phase ψ=0°. The M2

signal in microgals at any time, t, can be generated from the expression

$$\Delta g_{M2}\,(\phi,\lambda,t) = -\delta\,\frac{6x10^{8}}{a}\,\sqrt{5/96\pi}\;g\,C_{M2}\,\cos^{2}(\phi) \cdot \cos\,(\bar{n}\cdot\bar{\beta} + 2\lambda + \psi) \qquad (9)$$

where $\bar{\beta}$ was defined earlier, λ is east longitude, $\bar{n}=(2,0,0,0,0,0)$, g is magnitude of gravity in m/sec^2, a is the Earth semimajor axis in meters, and ϕ is latitude. The numerical value for C_{M2} is taken from Cartwright and Edden (1973). C_{M2} is equal to 0.63192. Differing sign conventions for the local phase angle, ψ, are found in the literature. For this reason (9) is explicitly shown. When the phase angle, ψ, takes on negative values, it is regarded as a lag.

The most precise tidal gravimeter results available are from the superconducting gravimeter studies of Warburton et al. (1975). These results are reported to have an accuracy of ±0.2% Tidal series with this gravimeter are available for La Jolla and Piñon Flat, Calif.

Tidal gravimeter results for the Australian stations Alice Springs and Canberra were taken from Melchior (1978). Results for Bruxelles were taken from Melchior et al. (1976). Observation results for Walferdange (Torge and Wenzel 1977), Potsdam (Altmann et al. 1977), and Mizusawa (Hosoyama 1977) were presented at the Eighth International Symposium on Earth Tides, Bonn, September, 1977.

The results of this study are given in table 1. The observations in terms of amplitude factor and local phase are given in columns A. The observed values are reduced by the theoretical solid Earth values δ=1.16 and ψ=0° and are given in columns B. Columns C show the predicted ocean tide contribution to the

Table 1.--Global tidal gravity

				A Observed		B Observed minus theoretical load[1]		C Computed ocean load[2]		D Observed minus computed load	
Station	Latitude (deg)	Longitude (deg)	Height (m)	Amplitude factor	Phase (deg)	Amplitude (μgal)	Phase (deg)	Amplitude (μgal)	Phase (deg)	Amplitude factor	Phase (deg)
La Jolla[3]	32.87	242.73	123.	1.1722	-3.33	3.64	-81.	3.35	-96.	1.1765	-0.25
								3.30	-96.	1.1764	-0.30
Piñon Flat[3]	33.59	243.54	1280.	1.1678	-1.29	1.42	-74.	1.74	-106.	1.1770	0.29
								1.74	-107.	1.1771	0.28
Alice Springs[4]	-23.72	133.83	590.	1.1656	-0.31	0.53	-48.	0.02	41.	1.1653	-0.29
								0.03	-146.	1.1660	-0.30
Canberra[4]	-35.32	149.00	663.	1.215	-2.02	3.57	-41.	2.37	-55.	1.1872	-0.37
								2.31	-56.	1.1879	-0.41
Bruxelles[5]	50.80	4.39	101.	1.1910	2.80	2.02	60.	2.04	66.	1.1663	-0.19
								2.00	66.	1.1662	-0.13
Walferdange[6]	49.62	6.15	295.	1.1910	2.61	1.81	55.	1.91	59.	1.1584	0.12
								1.87	59.	1.1588	0.17
Potsdam[7]	53.38	13.07	82.	1.1834	1.09	0.86	44.	1.30	38.	1.1450	-0.39
								1.28	38.	1.1456	-0.36
Mizusawa[8]	39.08	141.08	61.	1.1884	1.39	1.82	46.	2.26	56.	1.1599	-0.61
								2.20	56.	1.1606	-0.56

[1]Theoretical amplitude 1.16, phase 0°.
[2]Load deformation coefficients are taken from: Farrell (1972) for first line, Zschau (1978) (real part only) for second line.
[3]Warburton et al. (1975).
[4]Melchior (1978).
[5]Melchior et al. (1976).
[6]Torge and Wenzel (1977).
[7]Altmann et al. (1977).
[8]Hosoyama (1977).

gravity measurements using the Schwiderski M2 ocean tide model, the loading deformation coefficients of Farrell (1972) and Zschau (1978), and the integral Green's function technique presented in this paper. By comparing columns B and C, one can see that the predicted ocean contribution to the gravity signal agrees with the theoretical values (1.16,0°) to the 0.5-microgal level. These results are slightly better than the Australian results obtained by Bretreger and Mather (1978) using global ocean tide models of Hendershott and Zahel.

Columns D represent corrected amplitude factors and phases under the assumption that the predicted ocean tide contributions do indeed properly model the ocean load. Except for Piñon Flat and Walferdange, the phases seem to show a negative trend. The magnitude of these phases is contrary to that predicted by Zschau (1978) for an imperfectly elastic Earth. His modeling shows that the lag in gravity measurements should be very small (order of 1/1,000°). These corrected results should not be taken too seriously, however. These phases represent measurement accuracies of 0.5 microgal or less, which is not the case. They may also be subject to common calibration errors. Further improvement is also possible in modeling the ocean tide in the open oceans where direct measurements of the ocean tidal amplitude and phase are sparse.

Differences in Tidal Gravity

Because of the quality of the observations at La Jolla and Piñon Flat, further investigation is indicated as a result of the disagreement between the observed tidal values at these locations after correcting for the ocean contribution. The ocean predictions here seem to be slightly worse than those calculated by Warburton et al. (1975). Elimination of ocean tide effects from far afield is accomplished by subtracting the ocean effects between the two sets of observations. (La Jolla and Piñon Flat

are only 1° apart.) Table 2 shows the results of such differencing. One immediately notices that the differential predictions between Warburton et al. (1975) and the technique used here, along with the Schwiderski M2 model, are almost identical.

Table 2.--M2 ocean tidal differences in gravity between La Jolla and Piñon Flat

	Amplitude (μgal)	Phase (degree)
Observed	2.25	39.0
Goad (this study)	1.65	38.3
Warburton et al. (1975)	1.61	39.6

Western European Gravity Comparisons

Many tidal observations have been taken in western Europe during the past several years. Many of these observations were published by Melchior et al. (1976). Since then, Souriau (1979) published a set of corrections to the Melchior, Kuo, Ducarme observation set for the effects of the ocean tidal loading. His procedure was to use the Green's functions of Farrell (1972) in conjunction with ocean tidal information obtained from digitized cotidal charts at 0.25° spacing for the neighboring seas. Cotidal charts of several investigators were used to model the large water bodies. Table 3 gives these comparisons for western Europe. Notice that the predictions by Souriau and those computed using the Schwiderski M2 model with the load deformation coefficients of Zschau are very similar. The major differences occur at Bordeaux and Cambridge where the effects of the oceans are rather large. Obviously the 0.25° resolution of nearby seas was an important ingredient in the tidal corrections

Table 3.--Western European gravity

Station	Latitude (deg)	Longitude (deg)	Observed		Ocean load[1]		Ocean load[2]		Corrected[1]		Corrected[2]	
			Amplitude factor	Phase (deg)	Amplitude (μgal)	Phase (deg)	Amplitude (μgal)	Phase (deg)	Amplitude factor	Phase (deg)	Amplitude factor	Phase (deg)
Clermont-Ferrand	45.75	3.10	1.2092	3.60	3.02	69.	2.84	72.	1.1768	-0.04	1.1830	0.09
Bordeaux	44.83	-0.53	1.2119	7.03	5.66	80.	4.88	84.	1.1754	0.07	1.1900	0.95
Grasse	43.75	6.93	1.1884	2.13	1.98	61.	2.07	61.	1.1628	0.02	1.1583	-0.12
Strasbourg	48.58	7.77	1.1883	1.59	2.03	57.	1.87	54.	1.1543	-0.91	1.1545	-0.65
Walferdange	49.67	6.17	1.1910	2.61	2.15	60.	2.00	55.	1.1559	-0.24	1.1536	0.10
Witteveen	52.82	6.67	1.2136	2.15	1.98	49.	1.58	41.	1.1650	-0.41	1.1692	0.38
Bruxelles	50.80	4.37	1.1946	2.80	2.18	69.	2.02	63.	1.1669	-0.45	1.1659	-0.19
Hannover	52.38	9.70	1.1931	1.23	1.75	51.	1.67	46.	1.1534	-1.12	1.1515	-0.86
Graz	47.06	15.43	1.2120	1.12	1.41	40.	1.44	37.	1.1807	-0.10	1.1788	-0.06
Chur	46.85	9.53	1.1934	2.02	1.84	53.	1.78	51.	1.1615	0.02	1.1606	0.14
Torino	45.67	7.55	1.1975	1.35	2.05	58.	1.95	58.	1.1682	-0.94	1.1692	-0.84
Cambridge	52.20	0.12	1.1961	3.99	2.77	64.	2.54	53.	1.1510	-0.24	1.1388	0.57

[1]Souriau (1979).
[2]Goad (this study).

at these two sites. Nevertheless, predictions of this quality using a global M2 model without any modifications are very good.

CONCLUSIONS

Integrals of Green's functions are more suitable for the special case of tidal loading where the tide is represented by areas or cells of constant amplitude and phase. Their use allows for the inclusion of the height of the instrument above sea level. This method also directly uses sequences of load deformation coefficients which is advantageous for making comparisons. The 1° square Schwiderski M2 ocean tide model predicts accurately ocean load perturbations of Earth tidal gravity observations. Global calculations in this paper seem to be good to the 0.5-microgal level. Hopefully, with improvements in surface and space techniques, we are approaching a period when accurate checks between strain, tilt, gravity, and displacement observations will be possible for the prediction of effects of ocean tide models.

REFERENCES

Altmann, W., Dittfeld, H.-J., and Eistner, C., 1977: Results of gravimetric Earth-tide observations 1970-1976. Presented at the 8th International Symposium on Earth Tides, Institute for Theoretical Geodesy, University of Bonn, Sept. 19-24, 12 pp.

Bretreger, K. and Mather, R. S., 1978: Modeling ocean-loading effects on tidal gravity in Australia, Geophys. J. R. Astro. Soc. 52, 241-257.

Cartwright, D. E. and Edden, A. C., 1973: Corrected tables of tidal harmonics, Geophys. J. R. Astr. Soc., 33, 253-264.

Cartwright, D. E. and Tayler, R. J., 1971: New computations of the tide-generating potential, Geophys. J. R. Astr. Soc., 23, 45-74.

Farrell, W. E. , 1973: Earth tides, ocean tides and tidal loading, Phil Trans R. Soc. Lond A., 274, 253-259.

Farrell, W. E., 1972: Deformation of the Earth by surface loads, Rev. Geophys. Space Phys., 10, 761-797.

Hosoyama, K., 1977: On the tidal constant of gravity at Mizusawa. Proceedings of the 8th International Symposium on Earth Tides, Institut für Theoretische Geodäsie der Universitat Bonn. Sept., 19-24, 593-596.

Longman, I. M., 1963: A Green's function for determining the deformation of the Earth under Surface mass loads computations and numerical results, J. Geophys. Res., 68, 485-496.

Melchior, P., Kuo, J. T., and Ducarme, B., 1976: Earth tide gravity maps for western Europe, Phys. Earth Planet. Int., 13, 184-196.

Melchior, P., 1978: The Tides of the Planet Earth. Pergamon Press, Oxford, 609 pp.

Munk, W. H. and MacDonald, G. J. F., 1960: The Rotation of the Earth. Cambridge University Press, New York, 323 pp.

Pekeris, C. L., 1978: The bodily tide and yielding of the Earth due to tidal loading, Geophys. J. R. Astr. Soc., 52, 471-478.

Schwiderski, E. W., 1978: Global Ocean Tides Part 1: A Detailed Hydrodynamical Interpolation Model. U.S. Naval Surface Weapons Center-DL, TR-3866, Dahlgren, Va.

Souriau, M., 1979: Spatial analysis of tilt and gravity observations of Earth tides in western Europe, Geophys. J. R. astr. Soc., 57,585-608.

Torge, W. and Wenzel, H. G., 1977: Comparison of Earth tide observations with nine different gravimeters at Hannover. Proceedings of the 8th International Symposium on Earth Tides, Institut für Theoretische Geodäsie der Universtitat Bonn, Sept.19-24, 632-640.

Warburton, R. J., Beaumont, C., and Goodkind, J. M., 1975: The effect of ocean tide loading on tides of the solid Earth observed with the superconducting gravimeter, Geophys. J. R. Astr. Soc. 43, 707-720.

Zschau, J., 1978: Tidal friction in the solid Earth: loading tides versus body tides. Tidal Friction and the Earth's Rotation, Brosche and Sündermann (editors), Springer-Verlag, Berlin, 243 pp.

(Continued from inside front cover)

NOS NGS-5 National Geodetic Survey data: availability, explanation, and application. Joseph F. Dracup, June 1976, 45 pp (PB258475). The summary gives data and services available from from NGS, accuracy of surveys, and uses of specific data.

NOS NGS-6 Determination of North American Datum 1983 coordinates of map corners. T. Vincenty, October 1976, 8 pp (PB262442). Predictions of changes in coordinates of map corners are detailed.

NOS NGS-7 Recent elevation change in Southern California. S.R. Holdahl, February 1977, 19 pp (PB265-940). Velocities of elevation change were determined from Southern Calif. leveling data for 1906-62 and 1959-76 epochs.

NOS NGS-8 Establishment of calibration base lines. Joseph F. Dracup, Charles J. Fronczek, and Raymond W. Tomlinson, August 1977, 22 pp (PB277130). Specifications are given for establishing calibration base lines.

NOS NGS-9 National Geodetic Survey publications on surveying and geodesy 1976. September 1977, 17 pp (PB275181). Compilation lists publications authored by NGS staff in 1976, source availability for out-of-print Coast and Geodetic Survey publications, and subscription information on the Geodetic Control Data Automatic Mailing List.

NOS NGS-10 Use of calibration base lines. Charles J. Fronczek, December 1977, 38 pp (PB279574). Detailed explanation allows the user to evaluate electromagnetic distance measuring instruments.

NOS NGS-11 Applicability of array algebra. Richard A. Snay, February 1978, 22 pp (PB281196). Conditions required for the transformation from matrix equations into computationally more efficient array equations are considered.

NOS NGS-12 The TRAV-10 horizontal network adjustment program. Charles R. Schwarz, April 1978, 52 pp (PB283087). The design, objectives, and specifications of the horizontal control adjustment program are presented.

NOS NGS-13 Application of three-dimensional geodesy to adjustments of horizontal networks. T. Vincenty and B. R. Bowring, June 1978, 7 pp (PB286672). A method is given for adjusting measurements in three-dimensional space without reducing them to any computational surface.

NOS NGS-14 Solvability analysis of geodetic networks using logical geometry. Richard A. Snay, October 1978, 29 pp (PB291286). No algorithm based solely on logical geometry has been found that can unerringly distinguish between solvable and unsolvable horizontal networks. For leveling networks such an algorithm is well known.

NOS NGS-15 Goldstone validation survey - phase 1. William E. Carter and James E. Pettey, November 1978, 44 pp (PB292310). Results are given for a space system validation study conducted at the Goldstone, Calif., Deep Space Communication Complex.

NOS NGS-16 Determination of North American Datum 1983 coordinates of map corners (Second Prediction). T. Vincenty, April 1979, 6 pp (PB297245). New predictions of changes in coordinates of of map corners are given.

NOS NGS-17 The HAVAGO three-dimensional adjustment program. T. Vincenty, May 1979, 18 pp (PB297069). The HAVAGO computer program adjusts numerous kinds of geodetic observations for high precision special surveys and ordinary surveys.

NOS NGS-18 Determination of astronomic positions for California-Nevada boundary monuments near Lake Tahoe. James E. Pettey, March, 1979. Astronomic observations of the 120th meridian were made at the request of the Calif. State Lands Commission.

NOS NGS-19 HOACOS: A program for adjusting horizontal networks in three dimensions. T. Vincenty, July 1979, 13 pp. Horizontal networks are adjusted simply and efficiently in the height-controlled spatial system without reducing observations to the ellipsoid.

NOS NGS-20 Geodetic leveling and the sea level slope along the California coast. Emery I. Balazs and Bruce C. Douglas, September 1979, 23 pp. Heights of four local mean sea levels for the 1941-59 epoch in California are determined and compared from five geodetic level lines observed (leveled) between 1968-78.

NOS NGS-21 Haystack-Westford Survey. W. E. Carter, C. J. Fronczek, and J. E. Pettey, September 1979,

(Continued on inside back cover)

(Continued)

NOAA Technical Reports, NOS/NGS subseries

NOS 65 NGS 1 The statistics of residuals and the detection of outliers. Allen J. Pope, May 1976, 133 pp (PB258428). A criterion for rejection of bad geodetic data is derived on the basis of residuals from a simultaneous least-squares adjustment. Subroutine TAURE is included.

NOS 66 NGS 2 Effect of Geoceiver observations upon the classical triangulation network. R. E. Moose and S. W. Henriksen, June 1976, 65 pp (PB260921). The use of Geoceiver observations is investigated as a means of improving triangulation network adjustment results.

NOS 67 NGS 3 Algorithms for computing the geopotential using a simple-layer density model. Foster Morrison, March 1977, 41 pp (PB266967). Several algorithms are developed for computing with high accuracy the gravitational attraction of a simple-density layer at arbitrary altitudes. Computer program is included.

NOS 68 NGS 4 Test results of first-order class III leveling. Charles T. Whalen and Emery Balazs, November 1976, 30 pp (GPO# 003-017-00393-1) (PB265421). Specifications for releveling the National vertical control net were tested and the results published.

NOS 70 NGS 5 Selenocentric geodetic reference system. Frederick J. Doyle, Atef A. Elassal, and James R. Lucas, February 1977, 53 pp (PB266046). Reference system was established by simultaneous adjustment of 1,233 metric-camera photographs of the lunar surface from which 2,662 terrain points were positioned.

NOS 71 NGS 6 Application of digital filtering to satellite geodesy. C. C. Goad, May 1977, 73 pp (PB-270192). Variations in the orbit of GEOS-3 were analyzed for M_2 tidal harmonic coefficient values which perturb the orbits of artificial satellites and the Moon.

NOS 72 NGS 7 Systems for the determination of polar motion. Soren W. Henriksen, May 1977, 55 pp (PB274698). Methods for determining polar motion are described and their advantages and disadvantages compared.

NOS 73 NGS 8 Control leveling. Charles T. Whalen, May 1978, 23 pp (GPO# 003-017-00422-8) (PB286838). The history of the National network of geodetic control, from its origin in 1878, is presented in addition to the latest observational and computational procedures.

NOS 74 NGS 9 Survey of the McDonald Observatory radial line scheme by relative lateration techniques. William E. Carter and T. Vincenty, June 1978, 33 pp (PB287427). Results of experimental application of the "ratio method" of electromagnetic distance measurements are given for high resolution crustal deformation studies in the vicinity of the McDonald Lunar Laser Ranging and Harvard Radio Astronomy Stations.

NOS 75 NGS 10 An algorithm to compute the eigenvectors of a symmetric matrix. E. Schmid, August 1978, 5 pp (PB287923). Method describes computations for eigenvalues and eigenvectors of a symmetric matrix.

NOS 76 NGS 11 The application of multiquadric equations and point mass anomaly models to crustal movement studies. Rolland L. Hardy, November 1978, 63 pp (PB293544). Multiquadric equations, both harmonic and non-harmonic, are suitable as geometric prediction functions for surface deformation and have potentiality for usage in analysis of subsurface mass redistribution associated with crustal movements.

NOS 79 NGS 12 Optimization of horizontal control networks by nonlinear programing. Dennis G. Milbert, August 1979, 44 pp. Several horizontal geodetic control networks are optimized at minimum cost while maintaining desired accuracy standards.

NOAA Manuals, NOS/NGS subseries

NOS NGS 1 Geodetic bench marks. Lt. Richard P. Floyd, Septelber 1978, 56 pp (GPO# 003-017-00442-2) (PB296427). Reference guide provides specifications for highly stable bench marks, including chapters on installation procedures, vertical instability, and site selection considerations.